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Monte Carlo Generation of Two Body Resonant States*

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For Monte Carlo calculations of elementary particle processes, the transition rate integral can be written in terms of intermediate mass variables. Events generated in this way bear a weight factor consisting of the product of the squared transition amplitude and several two-body phase space factors. Importance sampling, using a Breit–Wigner density, can improve efficiency in the case that one or more mass variables of the transition density are well approximated by a Breit–Wigner resonance. With the further restriction that the resonance occurs close to threshold and decays to two final-state particles, we have improved the efficiency by sampling a probability density which is the product of a Breit–Wigner density times the associated two-body phase space factor.

I. INTRODUCTION

For Monte Carlo calculations of elementary particle processes, the transition rate integral can be written [1, 2] in terms of intermediate mass variables $\omega_1\omega_2 \cdots \omega_{n-1}$;

$$\Gamma(P \mid m_1 m_2 \cdots m_n) \propto \int d\omega_1 \cdots \int d\omega_{n-2} \cdot t,$$

$$t = |F|^2 \cdot R_2(P \mid m_1\omega_1) \prod_{i=1}^{n-2} 2\omega_i R_2(\omega_i \mid m_{i+1}, \omega_{i+1}),$$

$$\omega_{n-1} = m_n .$$

The two-body phase space factors are;

$$R_2(\omega \mid m_1 m_2) = ((\omega^2 - (m_1 + m_2)^2)(\omega^2 - (m_1 - m_2)^2))^{1/2}/(2\omega^2).$$

Monte Carlo event configurations can be generated by sampling values for the intermediate mass variables.

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Assume that one or more mass variables (ω for example) of the transition density can be approximated by a Breit–Wigner density;

$$b(\omega) = (\gamma/\pi) \cdot (\gamma^2 + (\omega - \omega_0)^2)^{-1}, \qquad -\infty < \omega < \infty.$$

The density $b(\omega)$ can be integrated. The resulting probability distribution function can be sampled by inversion. It is common practice to use random deviates ω from $b(\omega)$ for importance sampling of the transition density. Unfortunately, this does not account for the ω dependence of the product of two-body phase space factors. The efficiency of sampling can still be low.

As a further restriction assume that the resonance occurs near threshold and decays into final state particles of masses m_1 and m_2;

$$\underline{\omega} \lesssim \omega_0, \qquad \omega \to m_1 + m_2.$$

Near threshold, the phase space factor $R_2(P \mid m_1\omega_1)$ is approximately constant over the resonance region. The resonance associated two-body phase space factor $R_2(\omega \mid m_1 m_2)$ rises rapidly from zero at threshold. We are thus motivated to find an algorithm for efficient sampling of a probability density which consists of $b(\omega)$ times the resonance associated two-body phase space factor;

$$f(\omega) = 2\omega \cdot b(\omega) \cdot R_2(\omega \mid m_1 m_2), \qquad \underline{\omega} < \omega < \bar{\omega}.$$

Random deviates ω sampled from this density will then allow more efficient importance sampling of the transition density.

Section II of this note describes an algorithm for sampling of the density $f(\omega)$. Section III compares results obtained using $f(\omega)$ versus $b(\omega)$ for $\pi n \to \rho\Delta$ at 15 GeV.

II. Generation of Two-Body Resonant States

We note that the factor $2\omega \cdot R_2(\omega \mid m_1 m_2)$ rises rapidly from threshold $\omega = m_1 + m_2$ and approaches ω asymptotically. Thus we begin by showing that we can sample random numbers from the density;

$$f^*(\omega) = \omega \cdot b(\omega), \qquad \underline{\omega} < \omega < \bar{\omega}.$$

To sample $f^*(\omega)$ we scale the variable $x = (\omega - \omega_0)/\gamma$. The unnormalized density can be written;

$$f^*(x) = \gamma x/(1 + x^2) + \omega_0/(1 + x^2), \qquad \underline{x} < x < \bar{x}.$$

The distribution function is thus;

$$
\begin{aligned}
F^*(x) &= ((AG^*(x) + BH^*(x))/(A + B),\\
A &= \gamma \ln((1 + \bar{x}^2)/(1 + \underline{x}^2)),\\
B &= 2\omega_0(\tan^{-1}(\bar{x}) - \tan^{-1}(\underline{x})),\\
G^*(x) &= \ln((1 + x^2)/(1 + \underline{x}^2))/\ln((1 + \bar{x}^2)/(1 + \underline{x}^2)),\\
H^*(x) &= (\tan^{-1}(x) - \tan^{-1}(\underline{x}))/(\tan^{-1}(\bar{x}) - \tan^{-1}(\underline{x})).
\end{aligned}
$$

To sample $F^*(x)$ we sample $G^*(x)$ with frequency $A/(A + B)$ and sample $H^*(x)$ with frequency $B/(A + B)$. $G^*(x)$ and $H^*(x)$ can be sampled by inverting the distribution function. The inverse of $G^*(x)$ is:

$$x = ((1 + \underline{x}^2) \cdot \exp(N \cdot \ln((1 + \bar{x}^2)/(1 + \underline{x}^2))) - 1)^{1/2}.$$

The inverse of $H^*(x)$ is:

$$x = \tan(N \cdot \tan^{-1}(\bar{x}) + (1 - N) \cdot \tan^{-1}(\underline{x})).$$

After sampling x then $\omega = \omega_0 + \gamma \cdot x$.

Random numbers sampled from $f^*(\omega)$ allow $f(\omega)$ to be sampled by rejection. Letting $g(\omega) = f(\omega)/f^*(\omega)$ we have:

$$f(\omega)\, d\omega = g(\omega) f^*(\omega)\, d\omega = g(\omega)\, dF^*.$$

We note that $g(\omega)$ is bounded by 1. With ω sampled from $F^*(\omega)$ we accept this value if $N < g(\omega)$, or equivalently if,

$$(\omega^2 - (m_1 + m_2)^2)(\omega^2 - (m_1 - m_2)^2) > (N \cdot \omega^2)^2.$$

III. Conclusion

To investigate the relative advantage of sampling with $f(\omega)$ instead of $b(\omega)$ we programmed both for 15 GeV. $\pi n \to \rho\Delta \to p\pi\pi\pi$. The factor t was taken to be.

$$t = R_2(W \mid \omega_\rho\omega_\Delta) \cdot 2\omega_\rho b(\omega_\rho)\, R_2(\omega_\rho \mid m_\pi m_\pi) \cdot 2\omega_\Delta b(\omega_\Delta)\, R_2(\omega_\Delta \mid m_p m_\pi).$$

For fixed W the maximum of $t_b = t/(b(\omega_\rho)\, b(\omega_\Delta))$ was estimated as the maximum observed value for a run of 100 events. For sampling with $f(\omega)$ the maximum of $t_f = R_2(W \mid \omega_\rho\omega_\Delta)$ occurs at threshold $\omega_\rho = 2m_\pi$, $\omega_\Delta = m_p + m_\pi$. A sample of 1000 unit weight events was generated by each method.

The rejection efficiency for t_b was 0.057. The rejection efficiency for t_f was 0.894. The internal efficiency of the function $f(\omega)$ was 0.573 samples returned per three uniform random deviates. The t_f method generated unit weight events at a rate 10 times greater than the t_b method (3 sec versus 30 sec per 1000).

There is an important point with respect to this particular application. In realistic simulations of πn collisions we must include the Fermi momentum of the bound neutron. The total center of mass energy W then varies from one event to the next. In the case under discussion $f(\omega)$ has the additional advantage that the maximum value of t_f is easily calculated as $R_2(W \mid 2m_\pi, m_p + m_\pi)$. On the other hand, the maximum value of t_b depends on the product of three two-body phase space factors, and the maximum as a function of W is difficult to determine.

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References

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